Quadratic Optimal Cooperative Control Synthesis with Flight Control Application

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An optimal control-law synthesis approach is presented that involves simultaneous solution for two cooperating controllers operating in parallel. One controller's structure includes stochastic state estimation and linear feedback of the state estimates, while the other controller involves direct linear feedback of selected system output measurements. This structure is shown to be optimal under the constraint of linear feedback of system outputs in one controller. Furthermore, it is appropriate for flight control synthesis where the full-state optimal stochastic controller can be adjusted to be representative of an optimal control model of the human pilot in a stochastic regulation task. The method is experimentally verified in the case of the selection of pitch-damper gain for optimum pitch tracking, where optimum implies the best subjective pilot rating in the task. Finally, results from application of the method to synthesize a controller for a mutlivariable fighter aircraft are presented, and implications of the results of this method regarding the optimal plant dynamics for tracking are discussed.

Introduction

QUANTITATIVE handling qualities specifications are frequently unavailable in a useful form for high-authority multivarible flight control synthesis; and, for radically unconventional flight vehicles, such data may not yet even exist.

A methodology that would facilitate high-order augmentation synthesis while leading to good piloted performance and subjective evaluation was proposed in Ref. 1 and applied to the air-to-air tracking task in Ref. 2. In this approach, emphasis was placed on the ever-present parallel, cooperative structure of the pilot and flight control system, as depicted schematically in Fig. 1. Optimal control theory was employed in the control-law synthesis, as well as in modeling the pilot's equalization in the system. (Such a human operator modeling approach is patterened after that of Kleinman et al. in Ref. 3). The control solution so obtained was essentially a Nash solution to a linear-quadratic game[4] and required feedback of all system dynamic states in the optimal augmentation control law. In addition, a simplified model of the pilot's "controller" was used, ignoring the "pilot's" state-estimator dynamics in the synthesis of the augmenting controller. The inclusion of these dynamics would lead to theoretical difficulties if the global optimal solution is sought. This fact is a result of the inaccessibility of the (pilot's) state-estimator dynamics for feedback, thus requiring state estimation in *both* controllers, and this problem is unsolved to the author's knowledge. Practically, however, suboptimal controllers can lead to acceptable performance (the usual case), and this is the problem addressed here.

In this paper, two basic extensions to the above approach are presented. The first is the inclusion of state estimation in the pilot's equalization, and the second is the constraint that the augmentation control law consists of a linear combination of *selected* measurements. The latter point leads, of course, to a suboptimal but much more practical design. It will be shown how this additional (pilot) state-estimator dynamics affect the augmentation gains in this case, and an experimental validation of the method applied to select a simple pitch-damper gain will be presented. Finally, results will be presented on application of the method to a multi-input, statically unstable aircraft.

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Control Solution

The system dynamics are expressed in terms of the linear relation

$$\dot{\bar{x}}=A\bar{x}+B_p\bar{u}_p+B_A\bar{u}_A+D\bar{w} \quad E\{\bar{w}(t)\bar{w}^T(\tau)\}=W\delta(t-\tau) \tag{1}$$

where $\bar{x}\epsilon\mathcal{R}^n$, $\bar{u}_p\epsilon\mathcal{R}^\ell$ and $\bar{u}_A\epsilon\mathcal{R}^m$, and A, B_p, B_A, D are constant matrices of appropriate dimension. The vector $\bar{w}$ is a zero-mean Gaussian white-noise process with intensity W. Measurements or outputs available to the two "controllers" $\bar{u}_p$ and $\bar{u}_A$, as in Fig. 1, are

$$\begin{aligned}\bar{y}_p&=C_p\bar{x}+\bar{v}_p\\ \bar{y}_A&=C_x\bar{x}+C_u\bar{u}_p\end{aligned} \qquad E\{\bar{v}_p(t)\bar{v}_p^T(\tau)\}=V_p\delta(t-\tau) \tag{2}$$

respectively. The vector $\bar{v}_p$ is also a zero-mean Gaussian white-noise process (with intensity V_p) representing the error in the pilot's observation, and $\bar{y}_A$ are the measurements (selected) for feedback augmentation, assumed error-free. So C_p is essentially defined by the pilot's task, while C_x and C_u may be chosen in the solution process.

The control $\bar{u}_p$ represents the human operator's input, or control surface deflections associated with the pilot's stick input, for example; and the solution for $\bar{u}_p$ will be similar to the optimal control human operator model of Ref. 3. The control $\bar{u}_A$ is the augmentation system, constrained to be the direct feedback of preselected outputs, or

$$\bar{u}_A=G\bar{y}_A \tag{3}$$

where G is the matrix of control gains to be determined.

Solution for $\bar{u}_p$

The controller $\bar{u}_p$ minimizes the performance index J_p, the pilot's objective in the task, subject to modeled human limitations. Now J_p is taken to be

$$J_p=E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T(\bar{x}^TQ\bar{x}+\bar{u}_p^TR_1\bar{u}_p+\dot{\bar{u}}_p^TR_2\dot{\bar{u}}_p)\,dt\right\} \tag{4}$$

where $E\{\cdot\}$ again indicates the expected value operator, and the weighting matrices are $Q \geq 0$, $R_1 \geq 0$, $R_2 > 0$. Since the pilot controls the augmented aircraft, the minimizing control policy for $\dot{u}_p$ must be found subject to the dynamic constraint

$$\dot{\bar{x}} = (A + B_A G C_x)\bar{x} + (B_p + B_A G C_u)\bar{u}_p + D\bar{w} \qquad (5)$$

where G is the matrix of augmentation gains yet to be found. Now defining $\bar{\chi}^T = [\bar{x}^T \mid \bar{u}_p^T]$, the optimal control input is given by

$$\dot{\bar{u}}_p = K\hat{\bar{\chi}} \quad K = -R_2^{-1}[0 \mid I]P = [K_x \mid K_u] \qquad (6)$$

where $\hat{\bar{\chi}}$, the best estimte of $\bar{\chi}$, is obtained from the Kalman filter

$$\dot{\hat{\bar{\chi}}} = \left[\begin{array}{c|c} A + B_A G C_x & B_p + B_A G C_u \\ \hline 0 & 0 \end{array}\right]\hat{\bar{\chi}} + \left[\begin{array}{c} 0 \\ \hline I \end{array}\right]\dot{\bar{u}}_p + M[\bar{y}_p - \{C_p \mid 0\}\hat{\bar{\chi}}] \qquad (7)$$

$$M = \Sigma C_p^T V_p^{-1}$$

again seen to depend on G

Finally, P and Σ are obtained from their respective Ricatti equations:

$$\left[\begin{array}{c|c} A + B_A G C_x & B_p + B_A G C_u \\ \hline 0 & 0 \end{array}\right]^T P + \Phi \left[\begin{array}{c|c} A + B_A G C_x & B_p + B_A G C_u \\ \hline 0 & 0 \end{array}\right] \left[\begin{array}{c|c} Q & 0 \\ \hline 0 & R_1 \end{array}\right] - P\left[\begin{array}{c} 0 \\ \hline I \end{array}\right] R_2^{-1}[0 \mid I]P = 0 \qquad (8)$$

and

$$\left[\begin{array}{c|c} A + B_A G C_x & B_p + B_A G C_u \\ \hline 0 & 0 \end{array}\right]^T \Sigma + \Sigma \left[\begin{array}{c|c} A + B_A G C_x & B_p + B_A G C_u \\ \hline 0 & 0 \end{array}\right]^T + \left[\begin{array}{c|c} DWD^T & 0 \\ \hline 0 & V_m \end{array}\right] - \Sigma \left[\begin{array}{c} C_p^T \\ \hline 0 \end{array}\right] V_y^{-1}[C_p \mid 0]\Sigma = 0$$

Now, consistent with the human operator model,[3] the control input is modified to the (suboptimal) relation

$$\dot{\bar{u}}_p = K\hat{\bar{\chi}} + \bar{v}_m \qquad E\{\bar{v}_m(t)\bar{v}_m^T(\tau)\} = V_m \delta(t - \tau)$$

or

$$\dot{\bar{u}}_p = K_x \hat{\bar{x}} + K_u \hat{\bar{u}}_p + \bar{v}_m \qquad (9)$$

or, for scalar u_p,

$$\tau_n \dot{u}_p = -\bar{g}\hat{\bar{x}} - u_p + v_m'$$

where τ_n is the human's neuromuscular lag time constant, and $\bar{v}_m$ is a zero-mean Gaussian white-noise process with intensity V_m that represents the error contaminating the pilot's commanded control. Note that the time delay and predictor present in the model of Ref. 3 are not included here, however. It would significantly complicate the synthesis procedure, and due to work by Phatak, et al.,[5] it is not absolutely necessary. Properly "tuned," this model structure can exhibit the same dynamic characteristics as the Ref. 3 model.

Solution for $\bar{u}_A$

For the input $\bar{u}_A$, we wish to find the controller $\bar{u}_A$ (or gain G), as in Eq. (3), that minimizes the index of performance

$$J_A = E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T (\bar{\chi}^T Q_A \bar{\chi} + \dot{\bar{u}}_p^T F_2 \dot{\bar{u}}_p + \bar{u}_A^T F_1 \bar{u}_A)\,dt \right\} \qquad (10)$$

subject to the constraints of Eqs. (5-7). The augmentation may be synthesized to be "piloted-optimal" in the sense that its index of performance J_A may incorporate J_p, as in Refs. 1 and 2. This approach hypothesizes J_p to be correlated with the pilot subjective rating of the vehicle in a task. In this case, J_A may be expressed as

$$J_A = J_p + E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{u}_A^T F \bar{u}_A\,dt \right\}$$

Now, in solving for $\bar{u}_A$, we must include the dynamics of the (pilot's) state estimator, Eq. (7). In addition the plant dynamics, Eq. (5). Substituting Eq. (9) into the above two relations, and defining the augmented state vector $\bar{q}^T = [\bar{\chi}^T \mid \hat{\bar{\chi}}^T]$, we may write the system dynamics in the form

$$\dot{\bar{q}} = \bar{A}\bar{q} + \bar{B}\bar{u}_A + \bar{D}\tilde{w} \qquad (11)$$

with

$$\tilde{w}^T[\bar{w}^T \bar{v}_m^T \bar{v}_p^T] \qquad E[\tilde{w}(t)\tilde{w}^T(\tau)] = \bar{W}\delta(t - \tau)$$

$$\bar{A} = \left[\begin{array}{cc|cc} A & B_p & 0 & 0 \\ 0 & 0 & K_x & K_u \\ \hline & 0 & A + B_A G C_x - M C_p & B_p + B_A G C_u \\ M C_p & 0 & K_x & K_u \end{array}\right]$$

$$\bar{B}^T = [B_A^T \mid 0 \mid \cdots \mid 0]$$

$$\bar{D} = \left[\begin{array}{cc|c} D & 0 & 0 \\ 0 & I & 0 \\ \hline 0 & 0 & \\ 0 & I & M \end{array}\right]$$

where the abve matrices are seen to depend on the pilot's controller (K_x, K_u, and M) not a priori known.

The objective function J_A may now be written

$$J_A = E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T (\bar{q}^T \bar{Q}\bar{q} + \bar{u}_A^T F \bar{u}_A)\,dt \right\} \qquad (12)$$

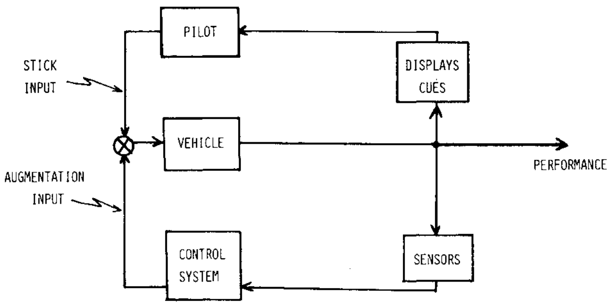

Fig. 1 Cooperative control schematic.

where, for example,

$$\bar{Q}=\begin{bmatrix} Q & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & K^T R_2 K \end{bmatrix}$$

Now the control law of the form

$$\bar{u}_A = G\bar{y}_A = G[C_A \mid 0]\bar{q}$$

including the gains G that minimize Eq. (12), subjet to Eq. (11), may be shown to be[6,7,8]

$$G=-F^{-1}\bar{B}^T HL\begin{bmatrix} C_A^T \\ \hline 0 \end{bmatrix}\left([C_A \mid 0]K\begin{bmatrix} C_A^T \\ \hline 0 \end{bmatrix}\right)^{-1} \quad (13)$$

with system covariance matrix $L=E\{\bar{q}\bar{q}^T\}$ satisfying the relation

$$[\bar{A}+\bar{B}G(C_A \mid 0)]L+L[\bar{A}+\bar{B}G(C_A \mid 0)]^T+\bar{D}\bar{W}\bar{D}^T=0 \quad (14)$$

and H satisfying

$$[\bar{A}+\bar{B}G(C_A \mid 0)]^T H+H[\bar{A}+\bar{B}G(C_A \mid 0)]+\bar{Q}$$

$$+\left[\begin{array}{c|c} C_A^T G^T F G C_A & 0 \\ \hline 0 & 0 \end{array}\right]=0 \quad (15)$$

An iterative numerical procedure is employed to simultaneously determine $\bar{u}_p$ and $\bar{u}_A$. The augmentation gain matrix G is chosen arbitrarily at first, and the pilot's control and filter gains [Eqs. (6) and (7)] are determined from Eq. (8). Then a solution for the augmentation gains G is obtained by solving Eqs. (13-15). The complete cycle, iterating on $\bar{u}_p$ and $\bar{u}_A$, is repeated until a stopping condition on the change in J_p and J_A between iterations is met.

It is important to note that, unlike the case of Levine and Athans,[6] this approach does not require the initial guess on G to stabilize the system, since that is not a requirement for determining a stabilizing $\bar{u}_p$ for a selected G. Furthermore, the optimal $\bar{u}_A$ for a problem may not even stabilize the plant. (For example, note that an aircraft spiral mode can be unstable and the vehicle may still have excellent dynamics and handling qualities). This is precisely why the controllers $\bar{u}_A$ and $\bar{u}_p$ must be determined simultaneously, and the conventional optimal control synthesis methods may not be appropriate and/or may be difficult to use.

By partitioning L into L_{11}, L_{12}, L_{21}, L_{22}, where $L_{11}=\text{cov}(\bar{x})$, $L_{22}=\text{cov}(\hat{x})$, etc., and by partitioning H likewise, we may obtain from Eq. (13);

$$G=-F^{-1}B_A^T[H_{11}L_{11}+H_{12}L_{12}]C_A^T[C_A L_{11} C_A^T]^{-1} \quad (16)$$

or

$$G=G_{11}+G_{12}$$

Note that if C_A^{-1} exists, or all plant states and pilot's control $\bar{u}_p$ are available for feedback, G_{11} is the standard optimal regulator gain. Further, regardless of the rank of C_A, G_{11} is the gain obtained if the state estimator is not included in the solution for $\bar{u}_p$ (i.e., if $\bar{v}_p$ is assumed zero), while G_{12} includes the dynamic coupling between the plant and pilot's state estimator (in H_{12} and L_{12}). It will be shown by the example below that G_{12} may be very significant.

Experimental Correlation

To evaluate the synthesis method, the optimal pitch-damper gain for pitch tracking will be determined and compared to experimental fixed-based simulation results. Although intended primarily for multivariable control synthesis, the method should yield good results in this simple case to have practical merit.

To be compatible with Ref. 9, the commanded pitch angle is governed by

$$\dot{\theta}_c=-0.1\theta_c+w; \qquad E[w(t)w(\tau)]=\sigma^2\delta(t-\tau)$$

σ^2 chosen to given $\sigma^2_{\theta_c}=10$ deg^2. For the aircraft considered (T-33), in level flight at Mach 0.5 and an altitude of 4545 m (15,000 ft), we obtain the following system dynamics in state variable form (all angles in degrees)

$$\dot{\bar{x}}=\begin{bmatrix} -0.1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1.0524 & -3.01 \\ 0 & 0 & 0 & -1.384 \end{bmatrix}\bar{x}$$

$$+\begin{bmatrix} 0 \\ 0 \\ -14.52 \\ -0.08 \end{bmatrix}\delta+\begin{bmatrix} w \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where only the short-period dynamics is modeled, and the state vector is $\bar{x}=[\theta_c, \theta, \dot{\theta}, \alpha]^T$. The total elevator input δ is the sum of pilot's and augmentation inputs, or $\delta_p+\delta_A$. The indices of performance to be minimized by δ_p and δ_A, respectively, are

$$J_p=E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T[(\theta_c-\theta)^2+r\dot{\delta}_p^2]\mathrm{d}t\right\}$$

$$J_A=J_p+E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T f\delta_A^2\mathrm{d}t\right\}$$

The weighting r on $\dot{\delta}_p$ (taken as $r=0.002$) was chosen to yield a neuromotor time constant τ_N [in Eq. (9)] of 0.1 s, and the weighting f on δ_A (taken as 0.01) was arbitrarily selected to be small compared to the unity weighting on $(\theta_c-\theta)$.

Although not usually the case, but for compatibility with Ref. 8, we assume here that the pilot is able to perceive only

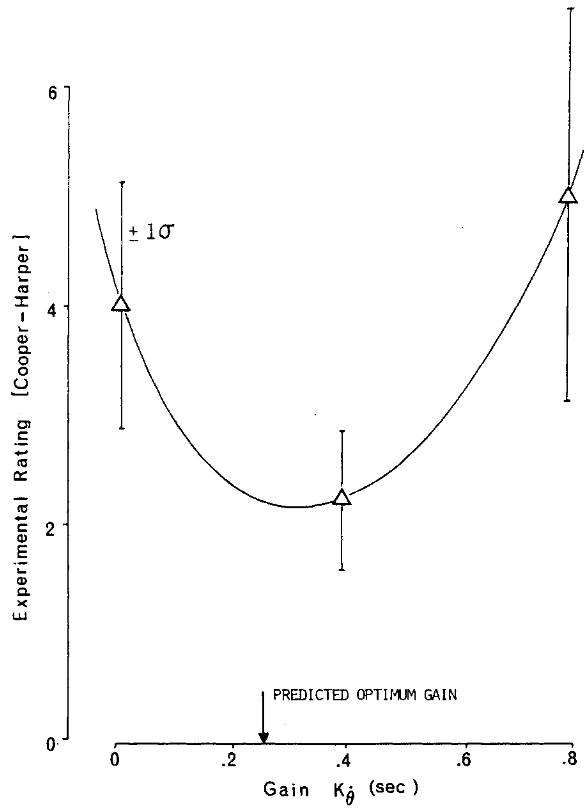

Fig. 2 Experimental correlation.

position error information $(\theta_c - \theta)$. The augmentation feedback variable y_A is pitch rate $\dot{\theta}$ only, thus

$$y_p = (\theta_c - \theta) + v_p \qquad C_p = [1.\ -1.\ 0\ 0]$$

$$y_A = [0\ 0\ 1.0]x$$

$$\delta_A = K_{\dot{\theta}}\dot{\theta}$$

The variances V_m and V_p on v_m and v_p are selected as in Ref. 3, i.e., chosen to yield an appropriate noise-to-signal ratio. Application of the method indicated the optimal $K_{\dot{\theta}}$ to be

$$K_{\dot{\theta}} = g_{11} + g_{12} = -0.09 + 0.33 = 0.24\ (1/\mathrm{s})$$

where g_{11} and g_{12} are from Eq. (16). As can be seen from Fig. 2, this gain and the pilot rating results obtained experimentally[10] are in very good agreement. So in this case, an augmentation system gain selected to minimize an appropriate cost metric as used in this method does indeed tend to yield "optimal" (lowest) subjective ratings quantified on the Cooper-Harper scale.[11] Finally, it is clear for this problem that ignoring the pilot's estimator dynamics, which would yield $g_{12} = 0$ above, would lead to erroneous results (i.e., $K_{\dot{\theta}} = -0.09$ instead of $+0.24$).

For reference, by comparing Figs. 2 and 3, we see the strong correlation between pilot ratings obtained experimentally and the sum of the mean-square error and stick rate—or the quadratic objective function—from simulation. All of these results are, furthermore, consistent with those of Ref. 9.

Augmenting a K/s^2 Plant

Attention is now focused on evaluating the method when applied to augmenting a plant frequently used in man-machine research. To be consistent with Refs. 1 and 12, in which this (K/s^2) plant was used, and Ref. 13, in which this

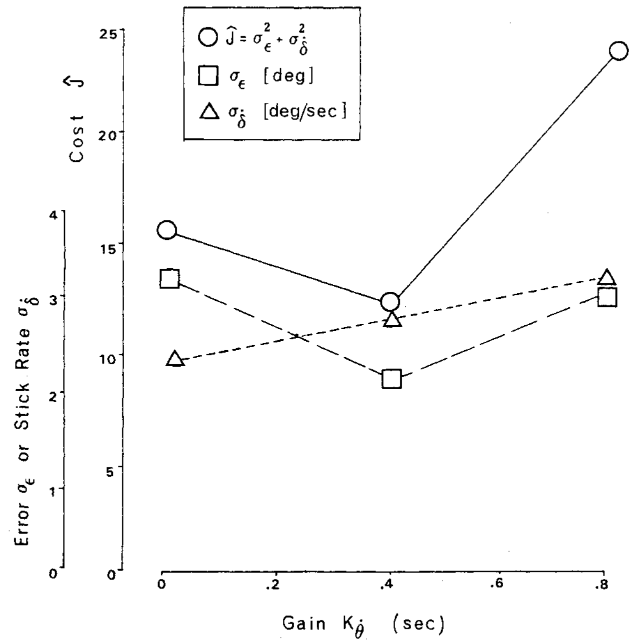

Fig. 3 Experimental rms performance.

and similar plants were evaluated experimentally, the task is that of tracking a displayed command signal θ_c, generated by

$$\ddot{\theta}_c + 3.\dot{\theta}_c + 2.25\theta_c = 3.67w(t) \qquad E\{w(t)w(\tau)\} = \delta(t-\tau)$$

Taking the plant gain K as 11.7 and defining the state vector $x^T = [\theta_c, \dot{\theta}_c, \theta, \dot{\theta}]$, we have

$$\dot{x} = \begin{bmatrix} 0 & 1. & 0 & 0 \\ -2.25 & -3. & 0 & 0 \\ 0 & 0 & 0 & 1. \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 11.7 \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 3.67 \\ 0 \\ 0 \end{bmatrix} w$$

where $\delta = \delta_A + \delta_p$, *the sum of the human operator's (pilot) and augmentation controls.*

The indices of performance to be minimized by δ_p and δ_A, respectively, are

$$J_p = E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T [(\theta_c - \theta)^2 + r\dot{\delta}_p^2]\,\mathrm{d}t \right\}$$

and

$$J_A = J_p + E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T f\delta_A^2\,\mathrm{d}t \right\}$$

The parameter r is again adjusted in such a way that a neuromuscular time constant [in Eq. (9)] $\tau_n = 0.1$ s is obtained, and the weighting f on the augmentation control is a free parameter that is varied to yield a family of controllers.

The human is usually assumed to be able to perceive both position error and error rate from the display,

Table 1 Augmentation results[a]

Control weighting, f	$(\theta_c-\theta)_{\text{rmp}}$	$\delta_{p_{\text{rms}}}$	$\dot{\delta}_{p_{\text{rms}}}$	δA_{rms}	J_p	PR	g_θ	g_θ	$g_{\dot\theta}$	ξ	$\dot\omega_n$
Unaugmented	1.17	1.00	1.3	...	1.86	8	0	0		...	0
10	0.79	0.66	1.1	0.08	0.77	5	−0.06	−0.03		0.19	0.9
1	0.63	0.49	1.0	0.23	0.46	4	−0.22	−0.08		0.30	1.6
0.1	0.62	0.61	1.0	0.48	0.45	4	−0.58	−0.13		0.30	2.6
0.01	0.61	0.80	1.0	0.66	0.44	4	−0.79	−0.15		0.30	3.0

[a] Units on θ's and δ's are in degrees; ω_n is in radians per second.

Table 2 Effect of estimator dynamics

Feedback variables,[a] $\bar{y}_A$		$(\theta_c-\theta)_{\text{rms}}$	$\delta_{p_{\text{rms}}}$	J_P	PR	g_θ	$g_{\dot\theta}$	ξ	ω_n
θ, $\dot\theta$	KF	0.63	0.49	0.46	4	−0.22	−0.08	0.3	1.6
	NKF	0.69	0.57	0.57	5	−0.11	−0.05	0.3	1.1
θ only	KF	0.76	0.66	0.71	5	−0.26	0	0	1.7
	NKF	0.86	0.79	0.93	6	−0.11	0	0	1.1

[a] Units on θ's and δ's are in degrees; ω_n is in radians per second.

Table 3 Flight condition specifications

Mach	$M=0.8$
Altitude	$h=23{,}000$ ft
Trim velocity	$U_0=819.57$ ft/s
Trim angle of attack	$\alpha_0=2.3454$ deg
Load factor	$1.0g$

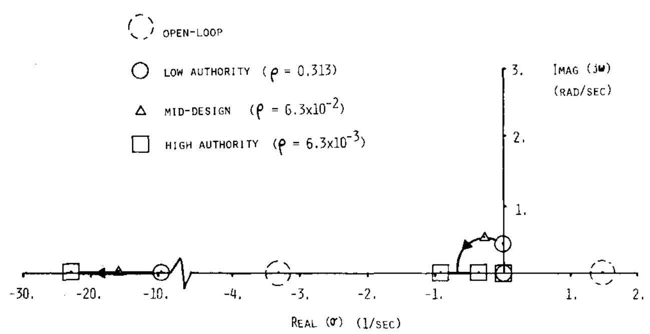

Fig. 4 Eigenvalue locus.

$$y_p=\begin{bmatrix}\theta_c-\theta\\ \dot\theta_c-\dot\theta\end{bmatrix}+v_p \qquad C_p=\begin{bmatrix}1. & 0 & -1. & 0\\ 0 & 1. & 0 & -1.\end{bmatrix}$$

The augmentation control law to be considered is

$$\delta_A=g_\theta\theta+g_{\dot\theta}\dot\theta \qquad C_A=[0\ \ 0\ \ 1.\ \ 1.]$$

although variations of this will be noted. The augmentation method was exercised, with covariances V_m and V_y selected to yield appropriate[3] noise-to-signal ratios.

In all the following results, a complete optimal control pilot model,[3] including time delay, attention sharing, etc., was used for rms performance evaluations. Furthermore, based on Refs. 1 and 12, a subjective "pilot rating" (PR) is predicted based on the index of performance J_p from this model.

Table 1 includes the model-based parametric results obtained, as a function of augmentation control weighting f. Shown are the rms tracking error $(\theta_c-\theta)$, augmentation activity δ_A, human's control activity and rate δ_p and $\dot\delta_p$,

Table 4 Augmentation results

rms error $(\theta_c-\theta)$	rms $\dot\delta_{\text{stick}}$	Cost J_p	Weighting r
0.68 deg	9.08 lb/s	0.74	1.610^{-3}

Control law: $\bar u_A=\begin{bmatrix}\delta_E\\ \delta_p\end{bmatrix}=G_\alpha\alpha+G_{\dot\theta}\dot\theta+G_\theta\theta$

$$G_\alpha=\begin{bmatrix}-0.208\\ -0.042\end{bmatrix} \quad \underset{\text{(in seconds)}}{G_{\dot\theta}=}\begin{bmatrix}0.777\\ 0.185\end{bmatrix} \quad G_\theta=\begin{bmatrix}0.211\\ 0.050\end{bmatrix}$$

Eigenvalues (augmented): ~0., −0.3±0.4j, −16. (1/s)

model-based performance J_p and PR, augmentation gains g_θ and $g_{\dot\theta}$, and and augmented plant dynamics with reference to the resulting plant transfer function:

$$\frac{\theta(s)}{\delta_p(s)}=\frac{11.7}{s^2+2\xi\omega_n s+\omega_n^2}=\frac{11.7}{s^2-11.7(g_{\dot\theta}s+g_\theta)} \tag{17}$$

Shown in Table 2 are similar results for cases not including the state estimator in the "pilot" control loop in the determination of the augmentation gains. These results, indicated as KF in the table, are shown to be inferior in performance $(\theta_c-\theta)$ and subjective rating (PR) to results obtained including the estimator dynamics (KF). This is true for the case with θ and $\dot\theta$ fed back in the augmentation ($y_A=\theta$, $\dot\theta$) as well as when θ alone is fed back ($y_A=\theta$ only). All these results (in Table 2) are for a control weighting f of unity, the value for which asymptotic performance and rating (in Table 1) appeared to be achieved. Again the significance of including the pilot's "filter" dynamics is noted.

Concerning the results of Table 1, it is significant that the augmented plant dynamics are type 0 rather than type 1 (K vs K/s at low frequency). Type 1 systems are known to have good low-frequency performance, since the closed-loop system frequency response tends to unity as $\omega\to 0$. Also, McRuer[14] and others have shown that the human in a compenstory manned tracking task adjusts his dynamic behavior such that the open-loop (including the human)

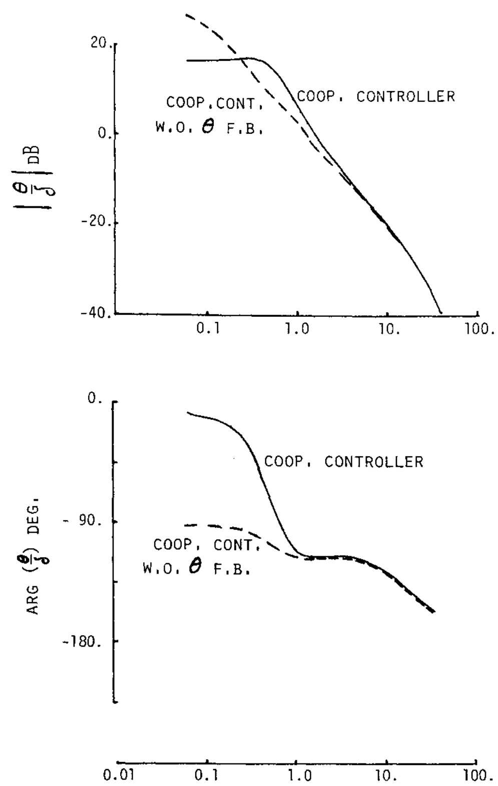

Fig. 5 Frequency response.

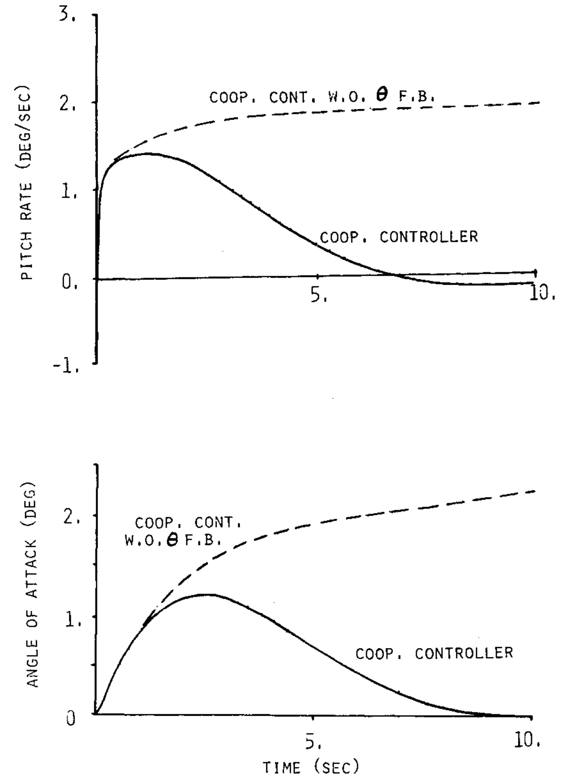

Fig. 6 Time responses.

dynamics approach K/s in the region of crossover frequency—thus yielding good closed-loop system stability characteristics. These facts, although not rigorous proof, form the basis for the rule-of-thumb that dynamics like K/s are "optimum" in tracking situations.

The differences depend on the definition of optimal. The method presented here indicates that the values of g_θ and $g_{\dot\theta}$ [see Eq. (17)] that minimize J_p (rms error and stick rate) do not yield K/s dynamics, or conversely, that for the unaugmented plant assumed, K/s-like dynamics do not minimize the rms error and stick rate in the task considered. [This is not to say that, given a different plant (e.g., $K<11.7$) or a different task (e.g., a lower-frequency command signal), that dynamics like K/s might indeed result.] In the minimization of J_p for the chosen tracking signal, low-frequency performance was sacrificed for an improvement in tracking error across the frequency range defined by the tracking command filter in selecting the optimal g_θ. And the result is a value of $g_\theta>0$. (Note, if $g_\theta=0$, a type 1 system results.)

Now if, on the other hand, K/s-like dynamics, and their associated advantages, are sought a priori, such dynamics may be obtained in three ways: 1) weight integral error in J_{aug}; 2) emphasize the low-frequency range by lowering the break frequency of the tracking command filter, for example; or 3) simply do not select θ for feedback. The latter alternative will guarantee type 1 augmented system dynamics and, at the same time, determine the "optimal" $g_{\dot\theta}$ (for a given weighting f) that minimizes error and stick rate for the chosen tracking task. Such dynamics have been experimentally evaluated in Ref. 13, for example, and can yield good performance and subjective rating. This approach, incidentally, will be explored below in the aircraft augmentation.

Finally, note that the results in Table 1 also indicate a reduction in static gain ($\sim 1/g_\theta$) or a reduced control sensitivity with augmentation. These results are consistent with those of Ref. 13, which indicated that a reduced plant gain ($K<11.7$) led to improved performance and rating, even for the K/s^2 plant. Also, although the control sensitivity is reduced, the method will never take the pilot completely "out of the loop" unless pilot stick deflection δ_{st} and command states (θ_c, $\dot\theta_c$) are available for feedback as in Ref. 1. In such a case, the controller is a complete tracking system rather than only augmenting plant dynamics.

Table 5 Augmentation without pitch attitude

rms error ($\theta_c-\theta$)	rms $\dot\delta_{\text{stick}}$	Cost J_p	Weighting r
0.68 deg	9.42 lb/s	0.77	$1.6\,10^{-3}$

Control law: $\bar u_A = \begin{bmatrix} \delta_E \\ \delta_p \end{bmatrix} = G_\alpha \alpha + G_{\dot\theta}\dot\theta$

$$G_\alpha = \begin{bmatrix} -0.047 \\ -0.019 \end{bmatrix} \quad \underset{\text{(in seconds)}}{G_{\dot\theta}=} \begin{bmatrix} 0.749 \\ 0.170 \end{bmatrix}$$

Eigenvalues (augmented): $-.05$, $+.03$, $-.62$, $-15.4(1/s)$

Multivariable Flight Control Application

We will now apply the technique to the augmentation synthesis of an aircraft similar to the AFTI/F-16 (Ref. 15). We will specifically have as the design objective that of optimizing pitch tracking performance. The vehicle state vector is taken as $\bar x_A^T=[u,\ \alpha,\ \dot\theta,\ \theta]$, the perturbation forward velocity, angle of attack, pitch rate, and pitch attitude angle. The control vector is $\bar u^T=[\delta_E,\ \delta_F]$, where δ_E is the elevator and δ_F the direct-lift flap deflection. (Note that in the following, the forward velocity u is nondimensionalized with the reference velocity U_0, and all the angular displacements and angular rates will have units of degrees and degrees per second, respectively.

Table 6 Control laws for comparison

Vehicle dynamics $\dot{\bar{x}} = A\bar{x} + B\bar{u}_A$

$$\bar{x}^T = [u, \alpha, \dot{\theta}, \theta]$$

Rate command control law: $$\bar{u}_A = \begin{bmatrix} \delta_E \\ \delta_F \end{bmatrix}_A = G_x\bar{x} + G_u\delta_{\text{stick}}$$

$$G_x = \begin{bmatrix} \sim 0 & 0.330 & 0.942 & 0.028 \\ \sim 0 & 0.073 & 0.232 & 0.007 \end{bmatrix} \quad G_u = \begin{bmatrix} -0.746 \\ -0.187 \end{bmatrix}$$

Eigenvalues (augmented): $-0.02 \pm 0.01j$, -0.98, -18.8 (1/s)

Air-to-air control law: $$\bar{u}_A = \begin{bmatrix} \delta_E \\ \delta_F \end{bmatrix} = L\bar{\xi} + Mx + N\delta_{st}$$

$$\dot{\bar{\xi}} = F\bar{\xi} + G\bar{x} + H\delta_{st}$$

$$L = \begin{bmatrix} 1. & 0. & 0.1056 \\ 0. & 0. & -1. \end{bmatrix} \quad M = \begin{bmatrix} 0.0007 & 0.5076 & 2.336 & 0. \\ 0. & 0. & 0. & 0. \end{bmatrix}$$

$$F = \begin{bmatrix} -1. & 1. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & -1. \end{bmatrix} \quad G = \begin{bmatrix} 0. & 0. & 7.66 & 0. \\ 0. & 0. & 7.67 & 0. \\ 0. & 0. & 0. & 0. \end{bmatrix}$$

$$N = \begin{bmatrix} -0.9308 \\ 0.24 \end{bmatrix} \quad H = \begin{bmatrix} -2.685 \\ -3.066 \\ 0.24 \end{bmatrix}$$

Eigenvalues (augmented): $\approx 0(2)$, -0.95, -38.8, $-1.(2)$, -3.6 (1/s)

Table 7 Performance comparison

Configuration	rms error, ϵ(deg)	rms stick rate, (lb/s) $\dot{\delta}_{\text{stick}}$ (lb/s)
Cooperative controller	0.68	9.08
Cooperative controller (without θ feedback)	0.69	9.42
Rate command	0.69	15.69
Air-to-air mode	0.69	24.02

The motion of the aircraft is referenced to the steady-state level flight condition given in Table 3.

The attitude command signal to be tracked, θ_c, is chosen consistent with the Neal-Smith study[16] and is generated by

$$\dot{\bar{x}}_c = \begin{bmatrix} \dot{\theta}_c \\ \ddot{\theta}_c \end{bmatrix} = \begin{bmatrix} 0 & 1. \\ -0.25 & -0.5 \end{bmatrix} \bar{x}_c + \begin{bmatrix} 0. \\ 1. \end{bmatrix} w$$

$$w = A_c\bar{x}_c + D_c w \qquad E\{w(t)w(\tau)\} = \sigma_w^2\delta(t-\tau)$$

The intensity σ_w^2 is chosen in this case to yield $\sigma_{\theta_c}^2 = 10.\ \text{deg}^2$.

The information perceived by the pilot, or his observation vector $\bar{y}_p$ is chosen to be

$$\bar{y}_p = \begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \theta \\ \dot{\theta} \end{bmatrix} + \bar{v}_p \qquad \epsilon = \theta - \theta_c$$

representative of a pursuit tracking task. Also the pilot's objective function for the pitch tracking task is taken as

$$J_p = E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T [(\theta_c - \theta)^2 + 0.063(\dot{\theta}_c - \dot{\theta}) + r\dot{u}_p^2]\,\mathrm{d}t \right\} \tag{18}$$

This selection of weightings, although different from the previous example, was chosen to be consistent with previous analytical and experiment data[17,18] developed for a variety of aircraft dynamics. The weighting on error rate (0.063) could be set to zero with little effect on these results. Likewise, the augmentation system objective function is

$$J_A = J_p + \left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{u}_A^T F \bar{u}_A\,\mathrm{d}t \right\} \tag{19}$$

with $F = \rho I > 0$ the augmentaion weighting matrix (I = identity). The parameter r [in Eq. (18)] is again adjusted to produce a pilot's neuromuscular lag time constant [Eq. (9)] $\tau_n = 0.1$ s.

The measurements selected in this analysis for augmentation feedback were $\bar{y}_A^T = [\alpha, \dot{\theta}, \theta]$, or we are performing the optimization assuming the control law

$$\begin{bmatrix} \delta_E \\ \delta_A \end{bmatrix}_{\text{aug}} = G_\alpha\alpha + G_{\dot{\theta}}\dot{\theta} + G_\theta\theta$$

It should be noted that the augmentation measurements in this case do not include pilot control input, although this is admissible in the formulation. In this exploratory investigation, a constant set of stick gains were selected, the pilot control input was taken as stick force, and

$$u_p = \delta_{\text{stick}} \begin{bmatrix} \delta_E \\ \delta_F \end{bmatrix}_{\text{pilot}} = \begin{bmatrix} K_{E_{\text{st}}} \\ K_{F_{\text{st}}} \end{bmatrix} \delta_{\text{stick}}$$

where $K_{E_{\text{st}}} = 1.0$ deg/lb, and $K_{F_{\text{st}}} = 0.25$ deg/lb.

Finally, the (observation and motor) noise intensities (or V_y and V_m) are selected as in Ref. 3. An iterative procedure was used, beginning with the augmentaion synthesis with assumed V_m and V_y; then the augmented system dynamics were evaluated with the complete pilot model (with time delays and attention sharing) to verify that the covariances (V_m and V_y) utilized were consistent with a properly calibrated pilot model.

Application of the synthesis procedure with a parametric variation of ρ [in Eq. (18)] results in augmented vehicle dynamics for which the locus of eigenvalues is shown in Fig. 4. Asymptotic performance (in terms of predicted tracking error and pilot model cost J_p) was obtained for a value of $\rho \approx 6.3 \times 10^{-2}$, or the results labeled "mid-design" in Fig. 4. The performance, cost weighting on $\dot{\delta}_p$, and control gains for this case are given in Table 4. The frequency response ($\theta/\delta_{\text{stick}}$) is shown in Fig. 5, and the pitch-rate and angle-of-attack time response to a unit (1 lb) step stick input are shown in Fig. 6.

Note the absence of K/s-like behavior in the augmented dynamics, as has the case in the previous K/s^2 example. These dynamics appear to be more like a lightly damped attitude command (K) response at low frequency, with definite attenuation of frequencies above ~2 rad/s.

Consider now a case where only angle of attack α and pitch rate $\dot{\theta}$ are used in the control law (no θ feedback). Solving for

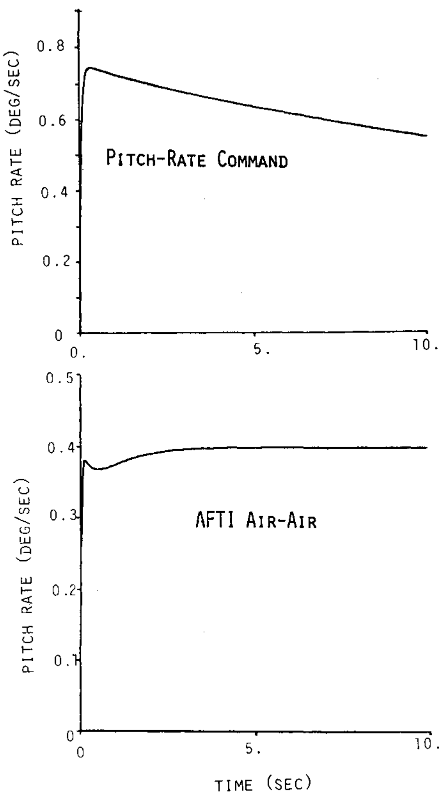

Fig. 7 Pitch-rate responses.

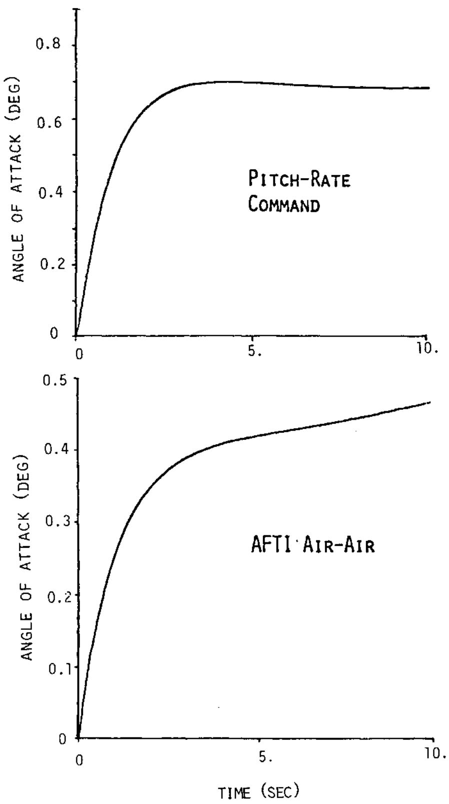

Fig. 8 Angle-of-attack responses.

the control gains in this case (again using $\rho = 6.3\times10^{-2}$) yields the results in Table 5, along with frequency and step response also shown in Figs. 5 and 6 for comparison. Clearly, in this case, the K/s behavior is present, and although not yielding the lowest cost J_p, the rms error and stick rate are only slightly worse than those in Table 4.

Observe, by comparing the control gains in Table 4 with those in Table 5, that the gains on pitch rate are very similar, but the gains on angle of attack are of opposite sign. The negative (α) gains in Table 4 would be effectively reducing the static stability of the already unstable airframe! This is allowable since the positive gains on pitch attitude prohibit large attitude divergence of the vehicle. This, of course, would preclude large excursions from the trim attitude, an undesirable characteristic for a conventional general-purpose control system, but perhaps not undesirable for a special-purpose fine-tracking augmentation system, the stated performance objective.

Control Systems Comparison

These output feedback control laws, or cooperative controllers, will be denoted CC and CC$(-\theta)$ in the following discussion, referring to the controller feeding back α, $\dot{\theta}$, and θ, and the case with no θ feedback, respectively. For purposes of comparison, we will consider two additional controllers, a generic pitch-rate command system and a control law similar to the standard air-to-air mode on the AFTI/F-16 (Ref. 15).

A rate command control system is chosen because it clearly leads to K/S dynamics. Briefly, it consists of an augmentation system that is designed to minimize the error between the aircraft pitch rate and the pilot's stick input, which is taken as the commanded pitch rate from the pilot.

In addition, an augmentation system similar to the air-to-air standard normal mode present in the AFTI/F-16 aircraft is simplified and linearized about the steady-state level flight condition of Table 3. This mode was also intended to provide precise tracking capabilities. The characteristics of these two controllers are listed in Table 6.

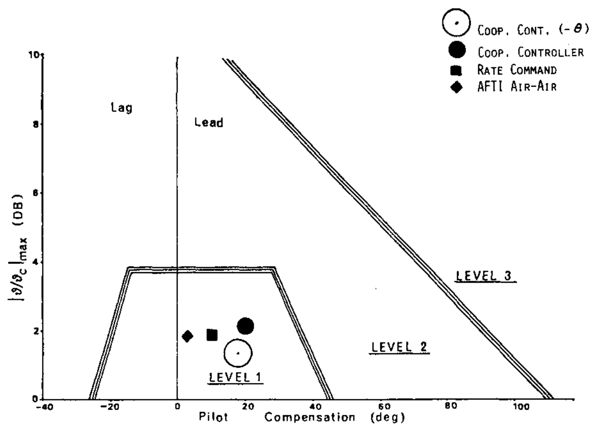

Fig. 9 Neal-Smith results.

Model-based predictions of "mission performance" are shown in Table 7 with time responses in Figs. 7 and 8. (The frequency responses of both these systems are like K/s.) Although the higher stick rates of the two comparison systems may be reduced with higher stick gains, it might be at the expense of higher tracking errors. It would then seem fair to state that the candidate designs exhibit equivalent predictd tracking performance scores at lower stick rates when compared to the rate command and air-to-air systems.

The eigenvalues of the two additional systems are included in Table 6, and all modes (in Tables 4-6) were identified from their eigenvectors. All four systems exhibit a relatively fast pitch-rate ($\dot{\theta}$) pole, with the cooperative controllers [CC and CC$(-\theta)$] both having an eigenvalue near $-16.$ (1/s), while other system's are at $-18.$ and $-35.$ (1/s), respectively.

Both the rate command and air-to-air systems have a real mode dominating the angle of attack near $-1.$ (1/s) and the traditional phugoid pair near the origin dominating the pitch and speed (or θ and u). [The air-to-air mode also has three control system roots, one at -3.5 (1/s) and two at $-1.$ (1/s).]

In definite contrast to these two systems, system CC (see Fig. 4) has a single root at the origin associated with velocity perturbation u. Then a complex coupled mode is present near $-0.3 \pm 0.4j$ that includes a significant amount of angle of attack α as well as attitude and velocity θ and u. (The participation of this mode in the angle of attack was clearly evident in the time histories). Clearly, with a higher frequency and a significant α participation, this mode is not a conventional phugoid mode. However, for system CC$-(\theta)$, the angle of attack was dominated by the single root at -0.6 (1/s), and two conventional phugoid roots were present. This is similar to the rate command and air-to-air systems.

As a final comparison, the Neal-Smith criterion[16] has been proposed to obtain pilot rating predictions from frequency response characteristics of the pilot-aircraft system in pitch tracking tasks. In their work, Neal and Smith hypothesized that "pilot rating is correlated with the pilot's compensation required to achieve good low frequency performance (good tracking) and the pilot/vehicle oscillation that resulted." Using this technique, a relationship was established between predicting pilot rating, pilot phase (lead or lag) compensation, and the resonance peak of the closed-loop system transfer function, or $|\theta/\theta_c|_{max}$. In recent studies by Bacon and Schmidt,[19] the same approach was extended with the use of the optimal control pilot model instead of describing the function modeling approach.

Based on the above, we compare the four configurations of interest here in terms of the Neal-Smith parameters, and results are given in Fig. 9. The levels of handling qualities are defined by: level 1 = 1.0-3.5 (Cooper-Harper scale, good); level 2 = 3.5-6.5 (Cooper-Harper scale, fair); level 3 = 6.5-10.0 (Cooper-Harper scale, unacceptable). From Fig. 9, all the configurations are seen to fall within the bounds of level 1.

Conclusion

An optimal cooperative control-law synthesis approach was presented that involves the simultaneous solution for two controllers operating in parallel. In flight control applications, the parallel human and augmentation controllers must be compatible—hence the simultaneous determination of pilot-model and augmentation gains is desirable. Also, the significance of including the pilot-model state-estimator dynamics in the control synthesis was demonstrated. Furthermore, it was noted that it was not necessary for the (output feedback) augmenting contoller alone to stabilize the plant, a situation that may arise with certain selected feedback measurements, and a case that leads to existence and solution difficulties for other optimal single-controller techniques.

Significantly, the overall approach, including gain selection and the pitch-rating/quadratic cost correlation, was verified for a case involving selection of pitch-damper gain to achieve the best subjective rating in a pitch-tracking task. Fixed-base simulation results were the basis of this verification.

Analytical evaluations showed a favorable comparison between the augmentation systems obtained from this approach and systems obtained using two alternate methods. Reduced stick rates resulted for the "cooperative" controllers, while the same level of tracking performance was predicted for all the controllers. All were analytically predicted to be acceptable to the pilot in the task considered. It was noted that with pitch attitude fed back, the augmented vehicle dynamics were characterized by nonconventional modes. On the other hand, more conventional type 1 systems, with the K/s behavior at low frequency, resulted in the other cases. If K/s dynamics are desired a priori, they can be obtained with this approach. In either case, however, the systems leading to "optimum" piloted performance are the desire.

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